INDIAN SCHOOL DARSAIT

Class XII — **Mathematics Worksheet**

Worksheet # 7 Differentiability # 1

(Chapter – 5: Continuity & Differentiability)

CLASS WORK

	Differentiate the following with respect to x
1.	i) $\text{Sin}^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ ii) $\text{Sin}^{-1}\left(3x-4x^3\right), -\frac{1}{2} < x < \frac{1}{2}$ iii) $\text{Cos}^{-1}\left(4x^3-3x\right), -\frac{1}{2} < x < \frac{1}{2}$
2.	i) $\text{Tan}^{-1}\left(\frac{2x}{1-x^2}\right), -1 < x < 1$ ii) $\text{Sin}^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$ iii) $\text{Cos}^{-1}\left(\frac{1-x^2}{1+x^2}\right), -1 < x < 1$
3.	i) $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ ii) $\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$ iii) $\tan^{-1}\left(\frac{x^{1/3}+a^{1/3}}{1-(ax)^{1/3}}\right)$
4.	i) $\tan^{-1}\left(\frac{2a^x}{1-a^{2x}}\right)$ ii) $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$
5.	i) $\cot^{-1}\left(\frac{\sqrt{1-\sin x}}{\sqrt{1+\sin x}}\right)$ ii) $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$ iii) $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$
6.	Find $\frac{dy}{dx}$ if y = $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$
7.	Differentiate with respect to x :- i) $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ $\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$
8.	Prove that $\frac{d}{dx}\left(\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)=\sqrt{a^2-x^2}$
9.	If $y=\sin^{-1}\left(x^2\sqrt{1-x^2}+x\sqrt{1-x^4}\right)$, prove that $\frac{dy}{dx}=\frac{2x}{\sqrt{1-x^4}}+\frac{1}{\sqrt{1-x^2}}$
10.	If $y=\tan^{-1}\left(\frac{5ax}{a^2-6x^2}\right)$, prove that $\frac{dy}{dx}=\frac{3a}{a^2+9x^2}+\frac{2a}{a^2+4x^2}$
11.	If $y=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\sqrt{\frac{a+b}{a-b}}\tan x/2\right)$, prove that $\frac{dy}{dx}=\frac{1}{a-b\cos x}$
12.	If $y=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\log\sqrt{1-x^2}$, prove that $\frac{dy}{dx}=\frac{\sin^{-1}x}{\left(1-x^2\right)^{3/2}}$
13.	If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$, then prove that $\frac{dy}{dx}=\frac{x}{\sqrt{1-x^4}}$
14.	Find $\frac{dy}{dx}$ for the following if: i) $x^2+xy+y^2=100$ ii) $x^{2/3}+y^{2/3}=a^{2/3}$ iii) $y^3-3xy^2=x^3+3x^2y$
15.	If $y\sqrt{1+x^2}=\log\left(\sqrt{1+x^2}-x\right)$, prove that $\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$
16.	If $x\sqrt{1+y}+y\sqrt{1+x}=0, x\neq y,$ prove that $\frac{dy}{dx}=\frac{-1}{\left(1+x\right)^2}$

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17.	If $\text{Sin}\, y = x\sin(a+y)$, s prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$
18.	If $\log(x^2+y^2) = 2\tan^{-1}\left(y/x\right)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$
19.	If $x\sqrt{1-y^2} + y\sqrt{1-x^2} = 1$, prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$
20.	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
21.	If $\sqrt{1-x^{2n}} + \sqrt{1-y^{2n}} = a\left(x^n - y^n\right)$, prove that $\frac{dy}{dx} = \frac{x^{n-1}}{y^{n-1}}\sqrt{\frac{1-y^{2n}}{1-x^{2n}}}$
22.	If $x^2 + y^2 = t - \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, prove that $\frac{dy}{dx} = \frac{1}{x^3 y}$
	HOME WORK
	Differentiate the following with respect to x :-
23.	i) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ ii) $\cot^{-1}\left(\sqrt{1+x^2}+x\right)$
24.	i) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ ii) $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ iii) $\tan^{-1}\left(\frac{a+b\tan x}{b-\tan x}\right)$ iv) $\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$
25.	i) $\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ ii) $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
26.	If y = $\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$, prove that $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$
27.	Find $\frac{dy}{dx}$ if y = $\tan^{-1}\left(\frac{2^{x+1}}{1-4^x}\right)$
28.	Differentiate: $\tan^{-1}\left(\frac{4x}{1+5x^2}\right)+\tan^{-1}\left(\frac{2+3x}{3-2x}\right)$ with respect to x
29.	If $\cos^{-1}\left(\frac{x^2+y^2}{x^2-y^2}\right) = \tan^{-1} a$, prove that $\frac{dy}{dx} = \frac{y}{x}$
30.	If $y = \cos^{-1}\left(\frac{x}{k}\right) - \frac{\sqrt{k^2-x^2}}{x}$, prove that $\frac{dy}{dx} = \frac{\sqrt{k^2-x^2}}{x^2}$
31.	Find $\frac{dy}{dx}$ for the following if: i) $\sin^2 y + \cos^2 y = 1$ $y + \sin y = \cos x$ $x^3 + y^3 = 3axy$ $\tan^{-1}\left(x^2+y^2\right) = a$

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32.	If $y\sqrt{1+x^2}=\log\left(\sqrt{1+x^2}-x\right)$, prove that $\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$
33.	If $\cos y = x\cos(a+y)$, prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$
34.	If $\sqrt{1-x^6}+\sqrt{1-y^6}=a\left(x^3-y^3\right)$, prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$
35.	If $\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=a$, prove that $\frac{dy}{dx}=\frac{x(1-\tan a)}{y(1+\tan a)}$
36.	If $\sqrt{y+x}+\sqrt{y-x}=a$, show that $\frac{dy}{dx}=\frac{2x}{a^2}$
37.	If $\sqrt{y+x}+\sqrt{y-x}=C$, show that $\frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2}{x^2}-1}$
	SELF STUDY
38.	Differentiate i) $\text{Tan}^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right]$ ii) $\text{Tan}^{-1}\sqrt{\frac{a-x}{a+x}}$ w.r.to x
39.	If $y=\text{Sin}^{-1}x+\text{Sin}^{-1}\sqrt{1-x^2}$, then prove that $\frac{dy}{dx}=0$
40.	Find $\frac{dy}{dx}$ if $ax^2+2hxy+by^2+2gx+2fy+c=0$
41.	If $x\text{Sin}(a+y)+\text{Sin}\,a\,\text{Cos}(a+y)=0$, prove that $\frac{dy}{dx}=\frac{\text{Sin}^2(a+y)}{\text{Sin}\,a}$
42.	If $y=\text{Sin}^{-1}\left[\frac{\sqrt{x}+1}{\sqrt{x}-1}\right]+\text{Sec}^{-1}\left[\frac{\sqrt{x}-1}{\sqrt{x}+1}\right]$, find $\frac{dy}{dx}$
43.	If $y=\text{Cos}^{-1}\left[\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right]$, find $\frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$
44.	If $y=\text{Sin}\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$, prove that $\frac{dy}{dx}=\frac{-x}{\sqrt{1-x^2}}$
45.	If $y=b\,\text{Tan}^{-1}\left[\frac{x}{a}+\tan^{-1}\left(\frac{y}{x}\right)\right]$, find $\frac{dy}{dx}$