INDIAN SCHOOL DARSAIT

Class XII — ***Mathematics Worksheet***

Worksheet # 1 Equivalence Relation

(Chapter – 1: Relations & Functions)

CLASS WORK	
1.	Show that the relation R = $\{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ defined on the set A = {1.2.3} is reflexive but neither symmetric nor transitive.
2.	Let A = {0, 1, 2, 3} and R be a relation on A defined by R = {(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)}. Is R reflexive, symmetric and transitive?
3.	Check whether the relation defined on the set $\{1,2,3,4,5,6\}$ by R = $\{(a,b): b=a+1\}$ is reflexive, symmetric and transitive.
4.	Let N be the set of natural numbers and R is a relation defined over N by R= $\{(x,y): x+2y=10\}$. Write the relation R and check if it is reflexive, symmetric and transitive.
5.	Show that the relation R = $\{(a,b): a \le b, a,b \in R\}$ is reflexive and transitive but not symmetric.
6.	Check whether the relation defined on the set of real numbers by R = $\{(a,b): a \le b^3\}$ is reflexive, symmetric and transitive.
7.	Let the relation R be defined on the set A = {1, 2, 3, 4, 5} by R = {(a , b): $\|a^2 - b^2\|$<8. Write the relation R. Also verify whether the relation is reflexive, symmetric and transitive.
8.	For the set A = 1, 2, 3} define a relation R on a by R = {(1,1), (2,3), (3,3), (1,3)}. Write the ordered pairs to be added to R to make it the smallest equivalence relation.
9.	Let A = {1, 2, 3} then write the following relations on A i) Reflexive and transitive but not symmetric ii) Symmetric but not reflexive and transitive iii) Reflexive, symmetric and transitive
10.	How many equivalence relations are possible on set A = {1, 2, 3}?
11.	Show that the relation R = $\{(a,b): \|a-b\| \text{ is even}\}$ defined on the set A = $\{1,2,3,4,5\}$ is an equivalence relation. Further show that the elements of $\{1,3,5\}$ are related to each other, the elements of $\{2,4\}$ are related to each other and no element of $\{1,3,5\}$ are related to any element of $\{2,4\}$.
12.	Show that the relation R = $\{(a,b): \|a-b\| \text{ is a multiple of } 4\}$ defined on the set A = $\{x \in Z, 0 \le x \le 12\}$ is an equivalence relation. Further find all the elements related to 1.
13.	Let R be a relation defined on the set A = $\{1,2,3,4,5,6,7\}$ by R = $\{(a,b): \text{both } a \text{ and } b \text{ are either even or odd}\}$. Show that R is an equivalence relation. Further show that i) all elements of $\{1,3,5,7\}$ are related to each other ii) all elements of $\{2,4,6\}$ are related to each other. iii) no element of $\{1,3,5,7\}$ is related to any element of $\{2,4,6\}$

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14.	Show that the relation $R=\{(a,b): a-b \, is \, divisible \, 3, a,b\in Z\}$ is an equivalence relation.
15.	Prove that the relation R on the set N×N defined by $(a,b)R(c,d)\Rightarrow$a+d = b+c, for all $(a,b),(c,d)\in$ N×N is an equivalence relation.
16.	Let A = {1,2,3,4,5,6,7} and R be relation in A×A by by $(a,b)R(c,d)$ if a + d = b + c for all (a,b), (c,d) ∈ A×A. Prove that R is an equivalence relation. Obtain the equivalence class of (2,5).
17.	Prove that the relation R on the set N×N defined by $(a,b)R(c,d)\Rightarrow$ad = bc, for all $(a,b),(c,d)\in$ N×N is an equivalence relation.
18.	Prove that the relation R on the set N×N defined by $(a,b)R(c,d)\Rightarrow$ad(b+c) = bc(a+d), for all $(a,b),(c,d)\in$ N×N is an equivalence relation.
19.	Let L be the set of lines in the XY plane and R be a relation defined on N defined by $R=\{(L_1,L_2): L_1 \parallel L_2\}$ is an equivalence relation. Find the set of all elements related to y = 2x + 4.
20.	Prove that the relation R on the set A of points in a plane given by $R=\{(P,Q): P \, and \, Q \, are \, equidis\tan t \, from \, the \, origin\}$ is an equivalence relation. Further show that the set of all points related to a point P ≠(0,0) is the circle passing through the point P with origin as centre.
21.	Determine whether the relation R defined on the set of real numbers given by $R=\{(a,b): a-b+\sqrt{3} \, is \; irrational\}$ is reflexive, symmetric and transitive.
HOME WORK	
22.	Let R = {(a, a^3): where a is prime number less than 5} is a relation. Find the range of R. Also verify whether the relation is reflexive, symmetric and transitive.
23.	Let R be a relation defined on the set N of natural numbers by R = {(x, y): 2x + y = 24, x, y ∈N}. find the domain and range of the relation. Check whether R is an equivalence relation.
24.	Check whether the relation defined on the set of real numbers by R = $\{(a,b): a<b^2\}$ is reflexive, symmetric and transitive.
25.	Let N be the set of natural numbers and R be a relation defined in N by $R=\{(a,b): a>b, a,b\in N\}$. Show that R is transitive but neither reflexive nor symmetric
26.	Show that the relation R = $\{(a,b): a=b\}$ defined on the set A = $\{x\in Z, 0\le x\le 12\}$ is an equivalence relation. Further find all the elements related to 1.
27.	Show that the relation R = $\{(a,b): 2 \, divides \, a-b\}$ defined on the set Z of integers is an equivalence relation.
28.	Let T be the triangles in a plane and R be relation in T given by R= $\{(T_1,T_2): T_1 \, is \, congruent \, to T_2\}$. Show that R is an equivalence relation.
29.	Prove that the relation R on the set N×N defined by $(a,b)R(c,d)\Rightarrow a^2+d^2=b^2+c^2$, for all $(a,b),(c,d)\in$ N×N is an equivalence relation.

30.	Prove that the relation R on the set Z of all integers defined by $(x, y) \in R \Rightarrow$ x − y is divisible by n is an equivalence relation.
31.	Let Z be the set of integers. Show that the relation $R = \{(a,b): a+b \text{ is even}, a,b \in Z\}$ is an equivalence relation.
32.	Prove that the relation R in the set A= {5,6,7,8,9} given by R= {(a, b): |a − b| is divisible by 2} is an equivalence relation. Obtain the equivalence class of 6.
33.	If R_1 and R_2 are two equivalence relations, prove that $R_1 \cap R_2$ is also an equivalence relation.
34.	In the set of natural numbers N define a relation R as follows: $\forall$n, m $\in$ N, nRm if on division by 5 each of the numbers n and m leaves the same remainder. Show that R is an equivalence relation. Also obtain the pair-wise disjoint subsets determined by R.
35.	If R is an equivalence relation on a set X, then show that R^{-1} is also an equivalence relation on X.
SELF STUDY	
36.	Show that the relation R on the set A of all triangles in a plane as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$ is an equivalence relation. Further consider three right triangles T_1 with sides 3,4,5, T_2 with sides 5,12,13 and T_3 with sides 6,8,10. Which triangles among T_1, T_2 and T_3 are related?
37.	Let S be the set of all students in a School with R as a relation in S given by $R = \{(S_1, S_2): S_1 \text{ and } S_2 \text{ are like minded students}\}$. Show that R is an equivalence relation i) Write two equivalence classes of S under the relation. (eg: set of honest students represented by [Honest Minded) For which equivalence class you would like to be a member
38.	Determine whether the relation R defined on the set of real numbers given by $R = \{(a,b): a,b \in R, a - b + \sqrt{2} \text{ is irrational}\}$ is reflexive, symmetric and transitive.
39.	Let f: X→Y defined by $f = \{(a,b): f(a) = f(b)\}$. Show that f is an equivalence relation
40.	Let P be the set of all points in a plane and R be a relation on P defined by $R = \{(A, B) \in P \times P: \text{if the distance between } A \text{ and } \mathbf{B} \text{ is less than } 3 \text{ units}\}$. Check whether R is an equivalence relation or not.
41.	Let Z be the set of all integers and R be a relation defined on Z by R= {(a, b): a − b is divisible by 5}. Show that R is an equivalence relation.
42.	Prove that the relation R defined on the set A = {1, 2, 3……12} given by R= {(a, b): |a − b| is divisible by 3} is an equivalence relation. Find the set of all elements related to 1.
43.	Show that the relation R defined on the set Z of integers given by R= {(a, b): 3 divides a - b} is an equivalence relation.
44.	Consider the relation R in the set of people in colony defined by aRb if a and b are members of joint family. Is R an equivalence relation?
45.	Let R be a relation defined on the set N of natural numbers defined by nRm if n divides m. Write whether R is reflexive, symmetric and transitive.

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46.	Show that the relation R defined on the set A of all polygons defined by $R = \{(P_1, P_2): P_1$ and P_2 have the same number of sides$\}$ is an equivalence relation. What is the set of all in A related to the right triangle T with sides 3,4 and 5?
47.	Show that the relation R defined on the set A of all books in a college library defined by $R = \{(x, y):$ x and y have the same number of pages$\}$ is an equivalence relation.
57	Let Z be the set of integers and Z_0 be the non-zero integers $= \{(a,b)(c,d): ad = bc\}$ be a relation on Z X Z_0, show that R is the equivalence relation on Z X Z_0.
58	Verify $S = \{(a,b) : a^2 + b^2 = 1\}$ is an equivalence relation on R or not.
59	Define $R = \left\{(x,y):\ x,y \in Q,\ x = \frac{1}{y}\right\}$ check the given relation is equivalence relation or not.