INDIAN SCHOOL DARSAIT

Class XII — ***Mathematics Worksheet***

Worksheet # 3 Binary Operations

(Chapter – 1: Relations & Functions)

CLASS WORK	
1.	State which of the following operations are binary? i) $a*b$ = a + ab, a, b ∈ Q ii) $a*b$ = a + 4b^2, a,b ∈ R iii) $a*b$ = a^3 + b^3, a, b ∈ N iv) $a*b$ = a – b + ab, a, b ∈ Z
2.	Check whether the following operations defined on the given set are commutative and associative: i) $a*b = \frac{a}{b+1}$, a,b ∈ R – {–1} iv) $a*b$ = 1, a,b ∈ N ii) $a*b = \frac{a+b}{2}$, a, b ∈ N iii) $a*b$ = a – b + ab, a, b ∈ Z
3.	On Q, the set of rational numbers, an operation $*$ is defined by $a*b=\frac{ab}{5}$ for all a, b ∈Q. Show that $*$ is i) a binary operation ii) commutative and associative. Find the identity element for $*$ in Q. Also prove that every non – zero element of Q is invertible
4.	Let $*$ be an operation on the set Q – {1}, defined by $a*b=a+b-ab$ for all a, b ∈Q – {1}. Check whether $*$ is commutative and associative. Find the identity element for with respect to $*$. Also prove that every element of Q – {1} is invertible?
5.	Let A = N∪{0} × N∪{0} and $*$ be a binary operation on A defined by (a,b) $*$ (c,d) = (a+c, b+d) for all (a,b) , (c,d) ∈ A. Show that $*$ is commutative and associative. Also find the identity element for $*$ in A.
6.	Let A = N × N and $*$ be a binary operation on A defined by (a, b) $*$ (c, d) = (ad + bc, bd) for all (a, b), (c, d) ∈ A. Show that i) * is commutative ii) * is associative iii) has no identity element
7.	Let * be a binary operation on N by a * b = LCM of a and b for all a, b∈N. i) Find 5*7, 20*16 ii) Is * commutative and associative? iii) Find the identity element in N w.r.to * iv) Which are the invertible elements of N?
8.	Let X be a non – empty set and * be a binary operation defined on P(X), the power set of X, defined by A*B = A∪B, for all A, B ∈P(X). i) Prove that * is commutative and associative ii) Find the identity element w.r.t * iii) Show that ϕ is the invertible element If O is another operation defined on P(X) by AOB = A ∩ B for all A, B ∈P(X). Show that * is distributive over O.
9.	Define a binary operation * on the set {0,1, 2, 3, 4, 5} as $a*b=\begin{cases}a+b, & if\ \ a+b<6\\ a+b-6, & if\ \ a+b\geq 6\end{cases}$. Show that i) 0 is the identity for this operation ii) each element of a is invertible with 6 – a is the inverse.

10.	Consider the binary operations *, o : R × R→R defined as a*b = |a – b| and aob = a for all a, b ∈R. Show that i) * is commutative but not associative ii) o is associative but not commutative iii) * is distributive over o
11.	Consider the binary operation $*$ on the set {1, 2, 3, 4, 5} defined by $a * b$ = HCF of a and b. i) Write the operation table. ii) Is $*$ commutative? iii) Also compute $(2*3)*5$ & $(2*3)*(4*5)$
12.	A binary operation * is defined on the set by $a*b=\begin{cases} a, & if \quad b=0 \\ \|a\|+b, & if \quad b\neq 0 \end{cases}$. If at least one of a and b is 0, then prove that $a*b=b*a$. Check whether $*$ is commutative. Also find the identity element w.r to $*$ if it exists.
13.	On the set M = A(x) = $\left\{\begin{bmatrix} x & x \\ x & x \end{bmatrix} : x\in R\right\}$ of 2×2 matrices, find the identity element for the binary operation "Multiplication of matrices". Also find inverse of each element of M.

HOME WORK

14.	Check whether the following operations defined on the given set are commutative and associative: - i) $a*b = 2^{ab}$, a,b ∈ Q ii) $a*b = a^3 + b^3$, a, b ∈ N iii) $a*b$ = ab + 1, ab ∈ Q
15.	Let $*$ be an operation onQ_0, the set of non – zero rational numbers, defined by $a*b=\frac{ab}{4}$ for all a, b ∈Q_o. Show that $*$ is i) a binary operation ii) commutative and associative. Find the identity element for $*$ in Q. What is the inverse of each element of Q_o?
16.	On the set R – {-1}, an operation $*$ is defined by $a*b=a+b+ab$ for all a, b ∈R – {-1}. Prove that $*$ is i) a binary operation ii) commutative as well as associative. Find the identity element for with respect to $*$. Also prove that every element of R – {-1} is invertible?
17.	Let $*$ be an operation on R_0, the set of non – zero real numbers, defined by $a*b=\frac{ab}{3}$ for all a, b ∈Q_o. Find the value of x, given that 2 $*$ (x $*$ 5) = 10
18.	Let R_0 be the set of all non – zero real numbers and A = $R_0 \times R_0$. Let $*$ be a binary operation on A defined by (a,b) $*$ (c,d) = (ac, bd) for all (a,b) , (c,d) ∈ A. i) Show that $*$ is commutative and associative ii) Find the identity element for $*$ in A iii) Find the invertible elements in A
19.	Let A = Q × Q and $*$ be an operation defined on A by (a,b) $*$ (c,d) = (ac, b+ad) for all (a,b) , (c,d) ∈ A. Determine whether $*$ is binary. If so find the identity element in A. Also find the invertible elements in A.

20.	Let X be a non – empty set and * be a binary operation defined on P(X), the power set of X, defined by $A*B = A\cap B$, for all A, B $\in$P(X). i) Prove that * is commutative and associative ii) Find the identity element w.r.t * iii) Show that X is the invertible element. If O is another operation defined on P(X) by AOB = $A \cup B$ for all A,B $\in$P(X). Show that * is distributive over O.
21.	Let X be a non – empty set and * be a binary operation defined on P(X), the power set of X, defined by $A*B = (A – B)\cup(B – A)$, for all A, B $\in$P(X). Prove that i) ϕ is the identity element w.r.t * in P(X) ii) A is invertible for all A $\in$P(X) and $A^{-1} = A$.
22.	Define a binary operation * on the set {0,1, 2, 3, 4, 5,6} as $a*b=\begin{cases} a+b, & if\ \ a+b<7 \\ a+b-7, & if\ \ a+b\geq 7 \end{cases}$. Show that i) Write the operation table ii) 0 is the identity for this operation iii) each element of a is invertible with 6 – a is the inverse.
23.	Define a binary operation * on the set A = {0,1, 2, 3, 4, 5} as a*b = ab(mod5). Show that i) 1 is the identity with respect to * ii) All elements of A are invertible with $2^{-1} = 3$ and $4^{-1} = 4$
24.	Let $*$ be a binary operation defined on the set Z of integers by $a*b=a+b-5$ for all a, b $\in$ Z. Show that $*$ is commutative and associative. Also find the identity element if it exists.
25.	Give an example of a binary operation which is i) commutative as well as associative ii) commutative but not associative iii) associative but not commutative
26.	Let $*$ be an operation defined on the set Z of integers by $a*b=a+b+2$ for all a,b $\in$ Z. i) Prove that $*$ is a binary operation. ii) Show that $*$ is commutative and associative. iii) Find the identity element w.r.t $*$ on Z iv) Find the inverse of a $\in$ Z.
SELF STUDY	
27.	Is $*$ defined on the set A = {1, 2, 3, 4, 5} by a $*$ b = LCM of a and b, a binary operation? Justify your answer.
28.	A binary operation * on R – {-1} defined as $a*b=\dfrac{a}{b+1}$. Is $*$ commutative and associative? Justify your answer.
29.	Consider the binary operation $*$ on the set A = {1, 2, 3, 4, 5} defined by a $*$ b = Min {a, b}. Write the operation table.
30.	Let $*$ be a binary operation defined on the set Q of rational numbers by $a*b=\dfrac{3ab}{5}$ Show that $*$ is commutative and associative. Also find the identity element if it exists.
31.	On the set Q_+ of all positive rational numbers define the operation $*$ by $a*b=\dfrac{ab}{3}$, a, b $\in Q_+$. i) Show that $*$ is a binary operation iii) Find the identity element w.r.t $*$ ii) Show that $*$ is commutative and associative iv) What is the inverse of a $\in Q_+$.

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32.	Consider the binary operation $*$ on the set A = {6, 7, 8, 9, 10} defined by $a * b = \text{Min}\{a, b\}$. Write the operation table.
33.	If $A = R - \{0\}$ and $*$ be a binary operation defined on A by $a*b = 2ab$, $\forall a,b \in A$. Then i) Show that $*$ is commutative ii) Show that $*$ is associative iii) Write the identity element w.r.t $*$ on A iv) If the inverse exists, find the inverse of a.