INDIAN SCHOOL DARSAIT

Class XII — **Mathematics Worksheet**

Worksheet # 15 Application of Derivatives # 5

Maxima & Minima # 1

(Chapter – 6 : Application of Derivatives)

CLASS WORK

Find local maximum and local minimum values of the function f given by

1.

i) $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

ii) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$

iii) $f(x) = \frac{-3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$

iv) $f(x) = 2x^3 - 6x^2 + 6x + 5$

v) $f(x) = x^3 - 6x^2 + 9x + 15$

vi) $f(x) = \text{Sin}\,x + \text{Cos}\,x, 0 < x < \frac{\pi}{2}$

vii) $f(x) = \text{Sin}\,x - \text{Cos}\,x, 0 < x < 2\pi$

viii) $f(x) = \text{Sin}^4 x + \text{Cos}^4 x, 0 < x < \frac{\pi}{2}$

2. Find two numbers whose sum is 24 and whose product is as large as possible.

3. Find two positive numbers x and y such that their sum is 35 and the product $x^2 y^5$ is a maximum.

4. Amongst all the pairs of positive numbers with product 256, find the numbers whose sum is the least.

5. Show that of all rectangles with given perimeter, the square has the greatest area

6. Show that of all rectangles inscribed in a given fixed circle, the square has the maximum area.

7. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.

8. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

9. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

10. An open box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

11. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

12. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

13. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

14. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

15. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\text{Sin}^{-1}\frac{1}{3}$

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16.	Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.
17.	Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$

HOME WORK

Find local maximum and local minimum values of the function f given by

18.	$f(x) = \text{Sin}\, x + \frac{1}{2}\text{Cos}\, 2x, \quad 0 \le x \le \frac{\pi}{2}$ $f(x) = \frac{2}{x} + \frac{x}{2}, x > 0$ $f(x) = \text{Sin}\, 2x - x, -\frac{\pi}{2} < x < \frac{\pi}{2}$
19.	Find two positive numbers x and y such that x + y = 60 and xy3 is maximum.
20.	Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum
21.	Show that of all rectangles with given area, the square has the least perimeter.
22.	A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is p m. Find the dimensions of the window to admit maximum light through the whole opening.
23.	A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?
24.	A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?
25.	Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetr find the dimensions of the can which has the minimum surface area?
26.	Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.
27.	Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.